Algorithmic Trading
Session 12
Performance Analysis III
Trade Frequency and Optimal Leverage

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Outline

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Introduction

Where Do We Stand in the Algo Prop Trading Framework?

- As we have seen, algorithmic proprietary trading strategies can be broken down into three subsequent steps: Signal Generation, Trade Implementation and Performance Analysis

- **Performance Analysis** is conducted after the trade has been closed and used in a backtesting context to judge whether the strategy is successful or not. In general, we can judge the performance according to five different metrics: return, risk, efficiency, trade frequency and leverage

- Sessions 10 -12 deal with the question of analyzing performance
  - **Session 10**: Performance Measurement
  - **Session 11:** Performance Analysis I: Returns, risk and efficiency
  - **Today's Session 12:** Performance Analysis II: Frequency of trades and leverage

Introduction

Performance Analysis

- Performance Analysis is a critical aspect of portfolio management. We split the analysis into five metrics: return, risk, efficiency, trade frequency and leverage
- Return is expressed as the geometric mean growth rate of the portfolio
- Risk is defined as the downside deviation of returns and can be expressed in terms such as standard deviation, downside standard deviation, maximum drawdown and length of maximum drawdown
- Efficiency measures set risk and return in relation. They are expressed through classical ratios such as Sharpe and Treynor measure, but also more modern ones such as the Sortino ratio. Win/Loss and Average Profit/Loss also indicate efficiency. A comparison to a benchmark is an indirect way of efficiency measurement as one targets a better return than the benchmark with similar risk or similar returns with lower risk
- Trade frequency is important to judge the impact of transaction costs and infrastructure requirements. The higher the trade frequency, the bigger the impact of transactions costs and requirement of a sophisticated infrastructure
- Leverage is another expression for money management. It deals with the question of which percentage of the total portfolio to invest in a given trade and how one can optimize this

Introduction

Review: Performance Drivers of Quantitative Trading Strategies

- Quantitative Investment Strategies are driven by **four success factors**: trade frequency, success ratio, return distributions when right/wrong and leverage ratio
- The higher the success ratio, the more likely it is to achieve a positive return over a one year period. Higher volatility of the underlying – assuming constant success ratio – will lead to higher expected returns
- The distribution of returns when being right / wrong is especially important for strategies with heavy long or short bias. Strategies with balanced long/short positions and hence similar distributions when right/wrong are less impacted by these distributional patterns. Downside risk can further be limited through active risk/money management, e.g. stop loss orders
- Leverage plays an important role to scale returns and can be seen as an "artificial" way to increase the volatility of the traded underlying. It is at the core of the money management question to determine the ideal betting size. For example, a 10 times leveraged position on an asset with 1% daily moves is similar to a full non-leveraged position on an asset with 10% daily moves

Trade Frequency
Classification

- **Time Frames:**
  - Long Term: Months to Years
  - Short Term: Days, Weeks, Months
  - Intraday: Seconds to Hours
  - High frequency: Fractions of Seconds

- **Hurdle Rate for Transaction Costs**
  - The more frequent the trading, the higher the transaction costs

- **Physical Infrastructure**
  - The more frequent the trading, the higher the requirement to have a sophisticated trading infrastructure

Trade Frequency

Infrastructure Requirements

- **Data feeds:** Although end of day data is available at low costs, tick by tick data can cost substantial amounts of money

- **Data Storage:** The more data you receive and the more frequent, the higher your data storage requirements. For every underlying you store, you should at least store OHLC for every day and Volume data. Next to this, you should also save information such as tickers, expiry dates, multiples, exchange on which the instrument is traded, time zone of the exchange, trading hours, currency and security type

- **Data Processing:** The more complex your trade signal generation process, the more computational power you will need. Additionally, the more time critical your investment strategy, you need to invest in computational power to minimize the time period between receiving the last information you need for your trade signal generation and trade implementation.

- **Backup Systems:** When you start out, a simple backup in the cloud or on an external hard disk will do. As more you grow and attract institutional clients, they will be very keen about your business contingency processes. Ideally you run the same system in parallel in two to three locations so that there is no impact on trading if one system fails

Optimal Leverage

Risk Management

- We'll discuss several methods of computing the optimal leverage that maximizes the compounded growth rate. Each method has its own assumptions and drawbacks. But, in all cases, we have to make the assumption that the future probability distribution of returns of the market is the same as in the past. This is usually an incorrect assumption, but this is the best that quantitative models can do. Even more restrictive, many risk management techniques assume further that the probability distribution of returns of the strategy itself is the same as in the past. And finally, the most restrictive of all assumes that the probability distribution of returns of the strategy is Gaussian

- As an institutional asset manager, client constraints limit the maximum drawdown of an account. In this case, the maximum drawdown allowed forms an additional constraint in the leverage optimization problem

- Also, it may be wise to avoid trading altogether during times when the risk of loss is high, hence setting leverage to close to 0. Therefore, many algorithmic trading strategies reduce exposure during times of increased expected volatility such as central bank announcements or significant macroeconomic numbers

- No matter how the optimal leverage is determined, the one central theme is that the **leverage should be kept constant.** This is necessary to optimize the growth rate whether or not we have the maximum drawdown constraint. Keeping a constant leverage may sound rather mundane, but can be counterintuitive when put into action

Optimal Leverage

Kelly Formula

- If one assumes that the probability distribution of returns is Gaussian, the Kelly formula gives us a very simple answer for optimal leverage ***f*****:** $f = m \, / \, s^2$ where m is the mean excess return and s^2 is the variance of the excess returns

- It can be proven that if the Gaussian assumption is a good approximation, then the Kelly leverage f will generate the highest compounded growth rate of equity, assuming that all profits are reinvested (see optional reading)

- However, even if the Gaussian assumption is really valid, we will inevitably suffer estimation errors when we try to estimate what the “true” mean and variance of the excess return are. And no matter how good one’s estimation method is, there is no guarantee that the future mean and variance will be the same as the historical ones. The consequence of using an overestimated mean or an underestimated variance is dire: Either case will lead to an overestimated optimal leverage, and if this overestimated leverage is high enough, it will eventually lead to ruin: equity going to zero

- On the other hand, the consequence of using an underestimated leverage is merely a submaximal compounded growth rate. Many traders justifiably prefer the later scenario, and they routinely deploy a leverage equal to half of what the Kelly formula recommends: the so-called half-Kelly leverage

Optimal Leverage
Simulated Returns

- If one relaxes the Gaussian assumption and substitutes another analytic form (e.g., Student's t) for the returns distribution to take into account the fat tails, we can still follow the derivations of the Kelly formula in Thorp's paper and arrive at another optimal leverage, though the formula won't be as simple anymore (This is true as long as the distribution has a finite number of moments, unlike, for example, the Pareto Levy distribution). For some distributions, it may not even be possible to arrive at an analytic answer. This is where Monte Carlo simulations can help

- The expected value of the compounded growth rate as a function of the leverage f is (assuming for simplicity that the risk-free rate is zero): $g(f) = \langle \log(1 + fR) \rangle$, where $\langle \dots \rangle$ indicates an average over some random sampling of the unlevered return-per-bar $R(t)$ of the strategy (not of the market prices) based on some probability distribution of R

- Even though we do not know the true distribution of R, we can use the so-called Pearson system to model it. model it. The Pearson system takes as input the mean, standard deviation, skewness, and kurtosis of the empirical distribution of R, and models it as one of seven probability distributions expressible analytically encompassing Gaussian, beta, gamma, Student's t, and so on. Of course, these are not the most general distributions possible. The empirical distribution might have nonzero higher moments that are not captured by the Pearson system and might, in fact, have infi nite higher moments, as in the case of the Pareto Levy distribution. But to capture all the higher moments invites data-snooping bias due to the limited amount of empirical data usually available. So, for all practical purposes, we use the Pearson system for our Monte Carlo sampling

Optimal Leverage

Optimizing Historical Growth Rate

- Instead of optimizing the expected value of the growth rate using our analytical probability distribution of returns one can of course just optimize the historical growth rate in the backtest with respect to the leverage. We just need one particular realized set of returns: that which actually occurred in the backtest
- This method suffers the usual drawback of parameter optimization in backtest: data-snooping bias. In general, the optimal leverage for this particular historical realization of the strategy returns won't be optimal for a different realization that will occur in the future. Unlike Monte Carlo optimization, the historical returns offer insufficient data to determine an optimal leverage that works well for many realizations
- Despite these caveats, brute force optimization over the backtest returns sometimes does give a very similar answer to both the Kelly leverage and Monte Carlo optimization

Optimal Leverage

Maximum Drawdown

- Most portfolio managers manage (at least in part) other people's assets, therefore maximizing the long-term growth rate is not the only objective. Often, their clients (or employers) will insist that the absolute value of the drawdown (return calculated from the historic high watermark) should never exceed a certain maximum. That is to say, they dictate what the maximum drawdown can be. This requirement translates into an additional constraint into our leverage optimization problem

- Unfortunately, this translation is not as simple as multiplying the unconstrained optimal leverage by the ratio of the maximum drawdown allowed and the original unconstrained maximum drawdown. Therefore, if the ideal leverage of say 40% results in a maximum drawdown of 50% and your drawdown constraint is 25%, you can not just reduce the leverage to 20%. The factor depends on the exact series of simulated returns, and so are not exactly reproducible

- In order to prevent future drawdowns from breaking the constraint, we can either use constant proportion insurance or impose a stop loss

Optimal Leverage
CPPI

- The often conflicting goals of wishing to maximize compounded growth rate while limiting the maximum drawdown have been discussed already. There is one method that allows us to fulfill both wishes: constant proportion portfolio insurance (CPPI)

- Suppose the optimal Kelly leverage of our strategy is determined to be f. And suppose we are allowed a maximum drawdown of $-D$. We can simply set aside D of our initial total account equity for trading, and apply a leverage of f to this subaccount to determine our portfolio market value. The other $1 - D$ of the account will be sitting in cash. We can then be assured that we won't lose all of the equity of this subaccount, or, equivalently, we won't suffer a drawdown of more than $-D$ in our total account

- If the trading strategy is profitable and the total account equity reaches a new high water mark, then we can reset our subaccount equity so that it is again D of the total equity, moving some cash back to the "cash" account. However, if the strategy suffers losses, we will not transfer any cash between the cash and the trading subaccount. Of course, if the losses continue and we lose all the equity in the trading subaccount, we have to abandon the strategy because it has reached our maximum allowed drawdown of $-D$. Therefore, in addition to limiting our drawdown, this scheme serves as a graceful, principled way to wind down a losing strategy

Optimal Leverage

Stop Loss

- There are two ways to use stop losses. The common usage is to use stop loss to exit an existing position whenever its unrealized P&L drops below a threshold. But after we exit this position, we are free to reenter into a new position, perhaps even one of the same sign, sometime later. In other words, we are not concerned about the cumulative P&L or the drawdown of the strategy

- The less common usage is to use stop loss to exit the strategy completely when our drawdown drops below a threshold. This usage of stop loss is awkward - it can happen only once during the lifetime of a strategy, and ideally we would never have to use it. That is the reason why CPPI is preferred over using stop loss for the same protection.

- Stop loss can only prevent the unrealized P&L from exceeding our selfimposed limit if the market is always open whenever we are holding a position. For example, it is effective if we do not hold positions after the market closes or if we are trading in currencies or some futures where the electronic market is open 24/5. Otherwise, if the prices “gap” down or up when the market reopens, the stop loss may be executed at a price much worse than what our maximum allowable loss dictates

- In some extreme circumstances, stop loss is useless even if the market is open but when all liquidity providers decide to withdraw their liquidity simultaneously as happened during the flash crash in May 2010.

Summary and Questions

- Performance Analysis is a critical aspect of portfolio management. We split the analysis into five metrics: return, risk, efficiency, trade frequency and leverage
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- Questions?

Sources

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