Algorithmic Trading
Session 2
Success and Risk Factors of Quantitative Trading Strategies

Oliver Steinki, CFA, FRM

Outline

- Introduction
- Performance Drivers of Quantitative Trading Strategies
- Basic Concepts
- Mathematical Expectation
- To Reinvest Trading Profits or not?
- What Is the Best Way to Reinvest?
- Kelly Criteria
- Finding the optimal f
- Summary and Questions

Contact Details: osteinki@faculty.ie.edu or +41 76 228 2794

Introduction

Risk / Money Management as a Performance Driver

- The performance of any trading strategy is **driven by the mathematical expectation of the strategy and the position size taken.** Obviously, position sizing (money management) is related to the size of your account level, but how exactly does it relate?
- We do not have control whether the next trade will be profitable or not as it is a mathematical expectations game. Yet, we do have control over the quantity we have on. Since one does not dominate the other, our resources are better spent concentrating on putting on the right quantity
- Risk management is about decision making strategies that aim to maximize the risk / return ratio within a given level of acceptable risk.
    - Example: Your client gives you a drawdown limit of 5%. Hence, you would like to optimize your trading strategy in terms of risk / return tradeoff, however it has to be done in a way that it must not breach the drawdown constraint
- Several concepts of money management will be explained by using gambling concepts. The mathematics of money management and the principles involved in trading and gambling are quite similar. The main difference is that in the math of gambling we are usually dealing with Bernoulli outcomes (only two possible outcomes), whereas in trading we are dealing with an entire probability distribution that the PnL may take

Performance Drivers

Performance Drivers of Quantitative Trading Strategies I

- Quantitative Investment Strategies are driven by **four success factors**: trade frequency, success ratio, return distributions when right/wrong and leverage ratio
- The higher the success ratio, the more likely it is to achieve a positive return over a one year period. Higher volatility of the underlying – assuming constant success ratio – will lead to higher expected returns
- The distribution of returns when being right / wrong is especially important for strategies with heavy long or short bias. Strategies with balanced long/short positions and hence similar distributions when right/wrong are less impacted by these distributional patterns. Downside risk can further be limited through active risk/money management, e.g. stop loss orders
- Leverage plays an important role to scale returns and can be seen as an "artificial" way to increase the volatility of the traded underlying. It is at the core of the money management question to determine the ideal betting size. For example, a 10 times leveraged position on an asset with 1% daily moves is similar to a full non-leveraged position on an asset with 10% daily moves

Performance Drivers

Performance Drivers of Quantitative Trading Strategies II

- Van Tharp introduced the concept of R multiple. 1 R measures the initial risk of a position, which is equal to the distance between entry and stop loss level. This assumes that one would be executed at stop loss level. Exit levels should be chosen so that the gains are higher than 1R. This is another way of saying cut losses short and let profits run. Example: Enter long position at 10 EUR with stop loss order at 9 EUR. 1R= initial risk = 10%

- In mathematical terms, the expected profit of a trading strategy is:

  Gain in %= frequency of trades * (winning % * avg. winning size – loosing % * avg. loosing size) * leverage ratio

  loosing % = 1 – winning % and avg. winning size = n * R, with n = average win to loss ratio

  Gain in % = frequency of trades * (winning % * n * R – (1 – winning %) * R) * leverage ratio * 1/100

- Example: A strategy trades daily, has a success ratio of 60%, equal average winning and losing size of 1 % and trades a leverage ratio of 200% of equity. In this case, the expected yearly gain is:

  Gain = 250 * (60% * 1 % – 40% * 1 %) * 2 = 100% p.a.

  Gain = 250 * (60% * 1/1 * 10% – (1 – 60%) * 10%) * 2 = 100% p.a.

Basic Concepts

- Ultimately, we have no control over whether the next trade will be profitable or not. Yet we do have control over the quantity we have on. Since one does not dominate the other, our resources are better spent concentrating on putting on the tight quantity
- The worst case loss on any given trade (which could be ensured through a stop loss, but does not necessarily have to) together with the level of equity in your account, should be the base for your position sizing, e.g. how many contracts to trade
- The divisor f of this biggest perceived loss is a number between 0 and 1 which determines how many contracts to trade. Assuming a portfolio of \$50.000, a worst case loss of \$5.000 per contract and a position of five contracts, this divisor is calculated as:

  \$50.000 / (\$5000 / f) = 5,

- Thus, you trade 1 contract per \$10.000 in equity with a **divisor f** of 0.5
- This divisor we will call by its variable name f. Thus, whether consciously or subconsciously, on any given trade you are selecting a value for f when you decide how many contracts or shares to put on.

Mathematical Expectation

- Mathematical Expectation (ME) is the amount you expect to make or lose, on average, each bet (trade)
- ME = $\sum[i = 1, N](P_i * A_i)$, with

  P = Probability of winning

  A = Amount won or lost

  N = Number of possible outcomes
- The mathematical expectation is computed by multiplying each possible gain or loss by its corresponding probability and then summing these products
- Exampe: Consider a game with a 50% chance of winning $2 and a (1-50%) = 50% chance of losing $1
- ME = (0.5*$2) + (0.5*(-$1)) = $0.5
- The expected profit per game is hence $0.5. This is a typical example of a game with an edge as the ME in a fair game should be $0.
- In a negative expectation game, there is no money management scheme that will create a winning strategy. If you continue to trade a negative expectation game, you will lose your entire stake in the long run.

Reinvest Trading Profits or Not?

- Reinvesting trading profits can turn a winning system into a losing system but not vice versa. A winning system becomes a losing system if returns are not consistent enough
- Changing the order or sequence of trades does not affect the final outcome, neither on a non reinvestment basis, nor on a reinvestment basis
- Reinvesting turns the linear growth function of a trading strategy with positive mathematical expectation into an exponential growth function
- The geometric mean is the best system to measure the tradeoff between profitability and consistency. It is calculated as the Nth root of the Terminal Wealth Relative (TWR). TWR represents the return on your stake as a multiple of your initial investment

  $TWR = \prod[1, N]HPR$, with

  HPR = Holding Period Returns

- For the three systems analyzed in excel, the TWRs and geometric means are as follows:
- The geometric mean expresses your growth factor per trade:

  $$G = \left(\frac{Final\ Stake}{Starting\ Stake}\right)^{Number\ of\ Trades}$$

System	TWR	Geometric Mean
System A	0.918	0.979
System B	1.071	1.017
System C	1.041	1.010

What Is the Best Way to Reinvest?

- So far, we have shown that reinvestment of returns leads to the highest geometric return and should therefore be used. We have implicitly assumed that we would reinvest at any time 100% of our equity. However, this is not ideal.
- Consider again our coin toss game: 50% chance of winning $2, 50% chance of losing 1%. What would be the ideal size to bet? If you bet 100% each time, you will be wiped out sooner or later. If you only bet $1 each time, you do not reinvest. Hence, the ideal betting size is somewhere in between these extremes
- Consider the ideal strategy for a negative expectancy game: You want to bet on as few trades as possible as your likelihood of losing increases with the number of trials.
  - Example: You are forced to play a game with 49% chance of winning $1, 51% chance of losing $1. The more often you bet, the greater the likelihood you will lose, hence you should only bet once.
- Returning to the positive expectancy game: The quantity f, that a trader can put on, lies between 0 and 1. f represents the trader's quantity relative to the perceived loss and total equity. As you know you have an edge over N bets, but not which bets will be losers/winners and by how much, the best way is to bet a constant percentage of your total equity. We are hence investigating the question of how to best exploit a positive expectation game?
- The answer: For an independent trials process, this is achieved by reinvesting a fixed fraction f of your total stake

Kelly Criteria

- The Kelly criteria deals with the question of optimal f in a gambling context. It states that we should bet that fixed fraction of our stake (f) which maximizes the growth function G(f) for events with two possible outcomes:

  $G(f) = P * \ln(1 + B + f) + (1 - P) * \ln(1 - f)$, with

  f = the optimal fixed fraction

  P = the probability of a winning bet

  B = ratio of amount won to amount lost if bet won/lost

  ln = the natural logarithm function

- Kelly formula applicable for events with equal wins and losses:

  $f = P - Q$, or $f = (2 * P) - 1$, which are equivalent, with

  Q = complement of P (1 – P)

  Example: Consider the following stream of bets: -1,+1,+1,-1,-1,+1,+1,-1,+1,+1

  f = 0.6 – 0.4 = 0.2 or (0.6 * 2) – 1 = 0.2

Kelly Criteria

- Kelly formula applicable for events with unequal wins and losses:

  $f = ((B+1) * P - 1)/B$, with notation as before

  Example: the two to one coin toss example

  f = ((2+1) * .5 – 1) / 2

  = (3 * 0.5 – 1) /2 = 0.5 / 2 = 0.25

- However, the Kelly formula is only applicable to outcomes that have a Bernoulli distribution. A Bernoulli distribution has only two possible discrete outcomes. Hence, the Kelly formula is applicable for gambling, but not for trading, where we have more than two possible outcomes and the Kelly formula would yield wrong results for the optimal f

- In case of gambling with a Bernoulli distributed outcome, the optimal f is:

  f = ME / B = 0.5 / 2 = 0.25 in the two to one coin toss example

Finding the Optimal f

- To find the optimal f for trading, we must amend our formula to find Holding Period Returns (HPR):

  $HPR = 1 + f \ * \left(\frac{-Trade}{Biggest\ Loss}\right)$, with

  f = the value we are using for f

  -Trade = the inverse of the PnL on any given trade (profits are negative, losses positive numbers)

  Biggest Loss = The PnL that resulted in the biggest loss (always negative)

  $$TWR = \prod[i = 1, N]\ (1 + f \ * \left(\frac{-Trade_i}{Biggest\ Loss}\right))$$

  $G = \prod[i = 1, N]\ (1 + f \ * (\frac{-Trade_i}{Biggest\ Loss})^{(1/N)})$, with notation as before and

  N = total number of trades

  G = Geometric mean of the HPRs

- We find the f that results in the highest G by looping through all possible f values starting at 0 in 0.01 increments and stop as soon as G starts decreasing as the f function has only one peak

Finding the Optimal f
Examples

- Consider the following sequence of trades: +9, +18,+7,+1+10,-5,-3,-17,-7
- Wrong Way: Kelly, f = 0.16 with a TWR of 1.085
- Correct way: f = 0.24 with a TWR of 1.096
- Comparison after x number of loops

# of loops	correct f TWR	wrong f TWR	Difference
1	1.096	1.085	1%
20	6.220	5.125	21%
50	96.506	59.453	62%
100	9,313.314	3,534.708	163%

- As you see, using the optimal f does not appear to offer much advantage over the short run, but over the long run it becomes more and more important. The point is, you must give the program time when trading at the optimal f and not expect miracles in the short run. The more time (i.e., bets or trades) that elapses, the greater the difference between using the optimal f and any other money-management strategy

Summary and Questions

- We do not have control whether the next trade will be profitable or not. Yet, we do have control over the quantity we have on. Since one does not dominate the other, our resources are better spent concentrating on putting on the right quantity
- The Kelly formula is only applicable to outcomes that have a Bernoulli distribution. It is a common mistake among traders to use it for trading
- Optimal f is the mathematical way to maximize the geometric mean of your trading system and hence the way to size positions if you want to exploit a positive expectancy game in the most efficient manner

- Questions?